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Anthropic Approach to the Cosmological Constant Problems

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There are now two cosmological constant problems: (i) why the vacuum energy is so small and (ii) why it comes to dominate at about the epoch of galaxy formation. Anthropic selection appears to be the only approach that can naturally resolve both problems. Here I review this approach, emphasizing the testable predictions that it makes for the dark energy density and for its equation of state.

KEY WORDS: cosmological constant; dark energy; anthropic selection.

1. INTRODUCTION

The cosmological constant problem is one of the most intriguing mysteries that we are now facing in theoretical physics. Until recently, there was only one cosmological constant problem and hardly any solutions. Now, within the scope of a few years, we have made progress on both accounts. We now have two cosmological constant problems (CCPs) and a number of proposed solutions. Here, I am first going to describe what the problems are and then comment on some of the solutions. My main focus will be on the anthropic approach, which I will argue is the only one that naturally resolves both CCPs.

Anthropic arguments are sometimes perceived as handwaving and unpredictive lore of questionable scientific value. One of my goals here is to dispel this notion. I am going to discuss several quantitative predictions that follow from the anthropic approach and which can soon be checked against upcoming observations. This review is based mostly on the work I did in collaboration with Jaume Garriga.

2. THE PROBLEMS

The cosmological constant is (up to a factor) the vacuum energy density, ρ_V. Particle physics models suggest that the natural value for this constant is set by the

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Planck scale M_P,

$$\rho_V \sim M_P^4 \sim (10^{18}\, GeV)^4, \tag{1}$$

which is some 120 orders of magnitude greater than the observational bound,

$$\rho_V \lesssim (10^{-3}\, eV)^4. \tag{2}$$

In supersymmetric theories, one can expect a lower value,

$$\rho_V \sim \eta_{SUSY}^4, \tag{3}$$

where η_{SUSY} is the supersymmetry breaking scale. However, with $\eta_{SUSY} \gtrsim 1$ TeV, this is still 60 orders of magnitude too high. This discrepancy between the expected and observed values is the first cosmological constant problem. I will refer to it as the old CCP.

Until recently, it was almost universally believed that something so small could only be zero, due either to some symmetry or to a dynamical adjustment mechanism. (For a review of the early work on CCP, see (Weinberg, 1989).) It therefore came as a surprise when recent observations (Perlmutter *et al.*, 1997, 1998; Riess *et al.*, 1998; Schmidt *et al.*, 1998) provided evidence that the universe is accelerating, rather than decelerating, suggesting a nonzero cosmological constant. The observationally suggested value is

$$\rho_V \sim \rho_{M0} \sim (10^{-3}\, eV)^4, \tag{4}$$

where ρ_{M0} is the present density of matter. This brings yet another puzzle. The matter density ρ_M and the vacuum energy density ρ_V scale very differently with the expansion of the universe, and there is only one epoch in the history of the universe when $\rho_M \sim \rho_V$. It is difficult to understand why we happen to live in this special epoch. Another, perhaps less anthropocentric statement of the problem is why the epoch when the vacuum energy starts dominating the universe ($z_V \sim 1$) nearly coincides with the epoch of galaxy formation ($z_G \sim 1$–3), when the giant galaxies were assembled and the bulk of star formation has occurred:

$$t_V \sim t_G. \tag{5}$$

This is the so-called cosmic coincidence problem, or the second CCP.

3. SOME PROPOSED SOLUTIONS

3.1. Quintessence and *k*-Essence

Much of the recent work on CCP involves the idea of quintessence (Caldwell *et al.*, 1998; Peebles and Ratra, 1988; Wetterich, 1988; Zlatev *et al.*, 1999). Quintessence models require a scalar field ϕ with a potential $V(\phi)$ aproaching zero at large values of ϕ. A popular example is an inverse power law potential,

$$V(\phi) = M^{4+\beta}\phi^{-\beta}, \tag{6}$$

with a constant $M \ll M_{\rm P}$. It is assumed that initially $\phi \ll M_{\rm P}$. Then it can be shown that the quintessence field ϕ approaches an attractor "tracking" solution,

$$\phi(t) \propto t^{2(2+\beta)}, \tag{7}$$

in which its energy density grows relative to that of matter,

$$\rho_\phi/\rho_{\rm M} \sim \phi^2/M_{\rm P}^2. \tag{8}$$

When ϕ becomes comparable to $M_{\rm P}$, its energy dominates the universe. At this point the nature of the solution changes: the evolution of ϕ slows down and the universe enters an epoch of accelerated expansion. The mass parameter M can be adjusted so that this happens at the present epoch.

A nice feature of the quintessence models is that their evolution is not sensitive to the choice of the initial conditions. However, I do not think that these models solve either of the two CCPs. The potential $V(\phi)$ is assumed to vanish in the asymptotic range $\phi \to \infty$. This assumes that the old CCP has been solved by some unspecified mechanism. The coincidence problem also remains unresolved, because the time of quintessence domination depends on the choice of the parameter M, and there seems to be no reason why this time should coincide with the epoch of galaxy formation.

A related class of models involves k-essence, a scalar field with a nontrivial kinetic term in the Lagrangian (Armendáriz-Picon *et al.*, 2000),

$$L = \phi^{-2}K[(\nabla\phi)^2]. \tag{9}$$

For a class of functions $K(X)$, the energy density of k-essence stays at a constant fraction of the radiation energy density during the radiation era,

$$\rho_\phi/\rho_{\rm rad} \approx {\rm const}, \tag{10}$$

and starts acting as an effective cosmological constant with the onset of matter domination. The function $K(X)$ can be designed so that the constant in Eq. (10) is $\lesssim 10^{-2}$, thus avoiding conflict with nucleosynthesis, and that k-essence comes to dominate at $z \sim 1$.

This is an improvement over quintessence, since the accelerated expansion in this kind of models always begins during the matter era. Galaxy formation can also occur only in the matter era, but still there seems to be no reason why the two epochs should coincide. The epoch of k-essence domination $z_{\rm V}$ is determined by the form of the function $K(X)$, and the epoch of galaxy formation $z_{\rm G}$ is determined by the amplitude of primordial density fluctuations, $Q = \delta\rho/\rho \sim 10^{-5}$. It is not clear why these seemingly unrelated quantities should give $z_{\rm V} \sim z_{\rm G}$ within one order of magnitude. And of course the old CCP also remains unresolved.

3.2. A Small Cosmological Constant From Fundamental Physics

One possibility here is that some symmetry of the fundamental physics requires that the cosmological constant should be zero. A small value of ρ_V could then arise due to a small violation of that symmetry. One could hope that ρ_V would be given by an expression like

$$\rho_V \sim M_W^8/M_P^4 \sim (10^{-3}eV)^4, \tag{11}$$

where $M_W \sim 10^3$ GeV is the electroweak scale. There have been attempts in this direction (Arkani-Hamed *et al.*, 2000; Frampton, 2000; Guendelman and Kaganovich, 2000; Kachru *et al.*, 1999), but no satisfactory implementation of this program has yet been developed.

An interesting idea based on braneworld models has been recently suggested by Dvali *et al.* (2002). They consider a brane in an infinite higher-dimensional bulk space, with supersymmetry broken on the brane, but not in the bulk. In models with more than two extra dimensions, they find that, surprisingly, the effective cosmological constant on the brane *decreases* as the brane tension is increased. A very small cosmological constant can be obtained when the brane tension is very large.

It is conceivable that the old CCP may eventually be resolved in this type of models, but even then, the time coincidence $t_V \sim t_G$ would still remain a mystery.

4. ANTHROPIC APPROACH

Both CCPs find a natural resolution in models where ρ_V is a random variable. The idea is to introduce a dynamical dark energy component X whose energy density ρ_X varies from place to place, because of stochastic processes that occured in the early universe. (Specific models for ρ_X will be discussed below.) The effective cosmological constant is then given by

$$\rho_V = \rho_\Lambda + \rho_X, \tag{12}$$

where $\rho_\Lambda \sim \eta_{SUSY}^4$ is the constant vacuum energy density. The cosmological constant problem now takes a different form. The question is not why ρ_Λ is very small (it is not), but why we happen to live in a region where ρ_Λ is nearly cancelled by ρ_X.

The key observation, due to Weinberg, is that the gravitational clustering that leads to galaxy formation effectively stops at $z \sim z_V$. An anthropic bound on ρ_V can be obtained by requiring that it does not dominate before the redshift z_{EG} when the earliest galaxies are formed. With $z_{EG} \sim 5$ one obtains (Weinberg, 1987)

$$\rho_V \lesssim - \rho_{M0}. \tag{13}$$

For negative values of ρ_V, a lower bound can be obtained by requiring that the universe does not recollapse before life had a chance to develop (Barrow and Tipler, 1986; Garriga and A. Vilenkin, 2002; Kallosh and Linde, 2002),

$$\rho_V \gtrsim -\rho_{M0}. \tag{14}$$

The bound (13) is a dramatic improvement over (1) or (3), but it still falls short of the observational bound by a factor of about 50. If all values in the anthropic range (13) were equally probable, then $\rho_V \sim \rho_{M0}$ would still be ruled out at a 90% confidence level. However, the values in this range are *not* equally probable.

The anthropic bound (13) specifies the value of ρ_V which makes galaxy formation barely possible. Most of the galaxies will be not in regions characterized by these marginal values, but rather in regions where ρ_V dominates after the bulk of galaxy formation has occured, that is

$$z_V \lesssim z_G. \tag{15}$$

Regions with $z_V \ll z_G$ will be rare, simply because they correspond to a very narrow range $|\rho_V| \ll \rho_{M0}$. Hence, we expect that a typical galaxy is located in a region where $z_V \sim z_G$. This explains the time coincidence (5) (Bludman, 2000; Garriga *et al.*, 2000). The expected value of ρ_V is thus

$$\rho_V \sim (1 + z_G)^3 \rho_{M0}. \tag{16}$$

The galaxy formation epoch z_G depends on the type of galaxy. In the standard cold dark matter cosmology, galaxy formation is a hierarchical process, with smaller objects merging to form more and more massive ones. We know from observations that some galaxies already existed at $z \sim 5$, and the theory predicts that some dwarf galaxies and dense central parts of giant galaxies could form as early as $z = 10$ or even $z = 20$. With such values of z_G, the dark energy density (16) would be far greater than observed. The agreement becomes much better if we assume that the conditions for civilizations to emerge arise mainly in giant galaxies, which form at low redshifts, $z_G \lesssim 1$. For $z_G \sim 1$, Eq. (16) gives (Efstathiou, 1995; Vilenkin, 1995) $\rho_V \sim 8\rho_{M0}$ and $\Omega_V \sim 0.9$. We shall improve on this rough estimate in the next section.

It is not clear how the condition $z_G \sim 1$ is to be justified. A suggesive observation is that we live in a disk of a giant spiral galaxy, and it is known that the galactic discs were assembled at $z \lesssim 1$ (Abraham and van der Bergh, 2001). But in any case, if the CCPs have an anthropic resolution, then, for one reason or another, the evolution of intelligent life should require conditions which are found mainly in giant galaxies, which completed their formation at $z \lesssim 1$. This is a prediction of the anthropic approach.

The analysis can be made more quantitative by introducing the probability distribution $\mathcal{P}(\rho_V)\, d\rho_V$, defined as being proportional to the number of independent observers who will measure ρ_V in the interval $d\rho_V$. This distribution can be represented as a product (Garriga and Vilenkin, 2002; Vilenkin, 1995)

$$\mathcal{P}(\rho_V) = \mathcal{P}_*(\rho_V) n_G(\rho_V) N_{obs}(\rho_V). \tag{17}$$

Here, $\mathcal{P}_*(\rho_V)\, d\rho_V$ is the prior distribution, which is proportional to the (comoving) volume of those parts of the universe where ρ_V takes values in the interval $d\rho_V$, $n_G(\rho_V)$ is the number of galaxies that form per unit comoving volume with a given value of ρ_V, and $N_{obs}(\rho_V)$ is the number of observers per galaxy.

The distribution (17) gives the probability that a randomly selected observer is located in a region where the effective cosmological constant is in the interval $d\rho_V$. If we are typical observers, then we should expect to observe a value of ρ_V somewhere near the peak of this distribution.

The prior distribution $\mathcal{P}_*(\rho_V)$ should be determined from the inflationary model of the early universe. Weinberg (1997, 2000a,b) has argued that a flat distribution

$$\mathcal{P}_*(\rho_V) = \text{const}, \tag{18}$$

should generally be a good approximation. The reason is that the function $\mathcal{P}_*(\rho_V)$ is expected to vary on some large particle physics scale, while we are only interested in its values in the tiny anthropically allowed range (13). Analysis shows that this Weinberg's conjecture is indeed true in a wide class of models, but one finds that it is not as automatic as one might expect (Garriga and Vilenkin, 2000a,b, 2002).

Of course, we have no idea how to estimate N_{obs}, but it seems reasonable to expect that it does not sensitively depend on ρ_V in the narrow range of interest, $N_{obs}(\rho_V) \approx \text{const}$. Then the distribution (17) reduces to

$$\mathcal{P}(\rho_V) \propto n_G(\rho_V). \tag{19}$$

The calculation of $n_G(\rho_V)$ is a standard astrophysical problem; it will be reviewed in the next section.

5. PREDICTIONS FOR Ω_V

Martel *et al.* (1998) (see also Efstathiou, 1995; Weinberg, 1997) presented a detailed calculation of $n_G(\rho_V)$ using the Press–Schechter formalism (Press and Schechter, 1974). They assumed a Gaussian density fluctuation field $\delta(\mathbf{x}, t)$ with a variance $\sigma(t)$ on the galactic scale ($M = 10^{12} M_\odot$),

$$P(\delta) \propto \left(-\frac{\delta^2}{2\sigma^2(t)}\right). \tag{20}$$

A galaxy forms when the linearized density contrast δ exceeds the critical value $\delta_c \approx 1.6$. The comoving density of galaxies can be found from

$$n_G(\rho_V) \propto P(\delta > \delta_c, t \to \infty) = \int_{\delta_c}^{\infty} d\delta P(\delta) = \mathrm{erfc}\left(\frac{\delta_c}{\sqrt{2}\sigma_\infty}\right), \tag{21}$$

where $\sigma_\infty \equiv \sigma(\infty)$

To estimate σ_∞ we note that prior to the dark energy domination, $t \ll t_V$, the scale factor behaves as $a(t) \propto t^{2/3}$, and the density fluctuations grow as $\sigma(t) \propto a(t) \propto t^{2/3}$. At $t > t_V$, the scale factor grows exponentially and the growth of density fluctuations freezes, $\sigma \approx \sigma_\infty$. Choosing the time of recombination t_{rec} as our reference time, we can write

$$\sigma_\infty \sim \sigma_{rec}\left(\frac{t_V}{t_{rec}}\right)^{2/3} \sim \sigma_{rec}\left(\frac{\rho_{rec}}{\rho_V}\right)^{1/3}. \tag{22}$$

Finally, substituting this to (21) and (19), we obtain

$$\mathcal{P}(\rho_V) \propto \mathrm{erfc}(y), \tag{23}$$

where

$$y = \frac{k}{\sigma_{rec}}\left(\frac{\rho_V}{\rho_{rec}}\right)^{1/3} \tag{24}$$

and $k \sim 1$. A careful analysis gives (Martel *et al.*, 1998) $k \approx 0.8$.

The distribution $d\mathcal{P}/d(\ln y)$ is a bell-shaped function with a maximum at $y \sim 1$ (see Fig. 1). More precisely, we expect $y > 0.79$ with probability

$$\mathcal{P}(y > 0.79) = 0.68 \quad (1\sigma \text{ confidence level}) \tag{25}$$

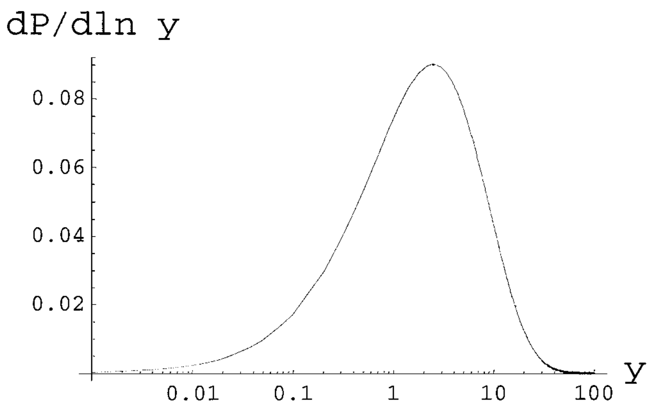

Fig. 1. The distribution (23).

and $y > 0.07$ with probability

$$\mathcal{P}(y > 0.07) = 0.95 \quad (2\sigma \text{ confidence level}). \tag{26}$$

Extracting predictions for the dark energy density parameter Ω_V from Eqs. (23) and (24) would be straightforward if we had an independent measurement of the density contrast at recombination, σ_{rec}. However, we do not presently have such a measurement. The value of σ_{rec} can be inferred from the CMB measurements, but for this task both the present value of Ω_V and the value of the Hubble parameter h would be needed. Thus, apart from the explicit dependence of y on Ω_V through $\rho_V/\rho_{rec} = (1 + z_{rec})^{-3}\Omega_V(1 - \Omega_V)^{-1}$, there is also an implicit dependence on Ω_V and h through σ_{rec}. The latter dependence can be obtained using the standard formulas given in (Liddle and Lyth, 2000).

The contour lines of the function $y(\Omega_V, h)$ corresponding to the 1σ and 2σ predictions (Garriga and Vilenkin, 2002) are shown in Fig. 2. (Note that the the notation Ω_D for the dark energy density in the figure differs from the notation Ω_V that we use in the text.) The excluded region lies to the left of the curves. We see that the currently favored values $\Omega_V \approx 0.7$, $h \approx 0.7$ are virtually excluded at the

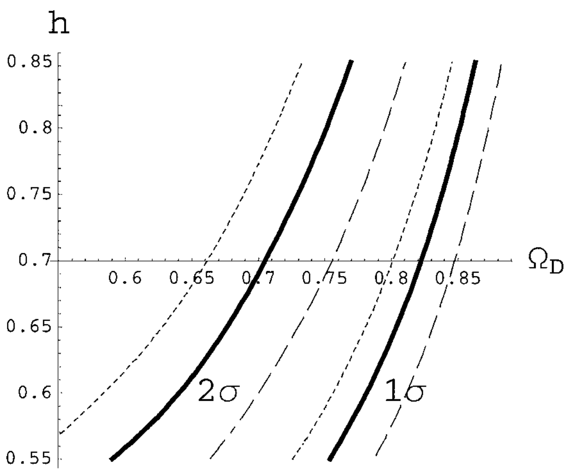

Fig. 2. Contours of the function $y(\Omega_V, h)$, corresponding to the 1σ (lower curves) and 2σ (upper curves) predictions. The excluded region lies to the left of the curves. The thick solid lines assume that the dominant contribution to N_{obs} is in galaxies of mass $M = 10^{12} M_\odot$. For comparison, we show the predictions for different choices of the mass. The short dashed curves correspond to the mass of the local group $M_{LG} = 4 \times 10^{12} M_\odot$, and the long dashed curves correspond to the mass of the bright inner part of our galaxy $M = 10^{11} M_\odot$. A scale invariant spectrum of density perturbations is assumed.

2σ level. Instead, the anthropic approach favors higher values of Ω_V and lower values of h.

The function $y(\Omega_V, h)$ has some dependence on the spectral index n of the fluctuation spectrum. A scale-invariant spectrum $n = 1$ is assumed in the figure, but the results are essentially unchanged if n is varied by a few percent. For example, for $n > 0.95$ and $h > 0.65$, the 1σ prediction is $\Omega_V > 0.79$.

The observational situation at this time is far from being clear. CMB and supernovae measurements yield $\Omega_D \approx 0.7$ (Percival *et al.*, 2002; Perlmutter *et al.*, 1997, 1998; Riess *et al.*, 1998; Schmidt *et al.*, 1998; Sievers *et al.*, 2002), while the observations of galaxy clustering give $\Omega_V \approx 0.8$ (Bahcall *et al.*, 2002).

6. MODELS WITH VARIABLE ρ_V

6.1. Scalar Field Models

As we saw in the preceeding sections, the anthropic approach naturally resolves both. CCPs, but it does require a particle physics model that would provide a dynamical dark energy component ρ_V and an inflationary cosmological model that would give a more or less flat prior distribution $\mathcal{P}_*(\rho_V)$ for $\rho_V = \rho_\Lambda + \rho_X$ in the anthropic range (13).

One possibility is that ρ_X is a potential $V(\phi)$ of some field $\phi(x)$ (Garriga and Vilenkin, 2000a). The slope of the potential is assumed to be so small that the evolution of ϕ is slow on the cosmological time scale. This is achieved if the slow roll conditions

$$M_P^2 V'' \ll \rho_V \lesssim \rho_{M0}, \tag{27}$$

$$M_P V' \ll \rho_V \lesssim \rho_{M0}, \tag{28}$$

are satisfied up to the present time. These conditions ensure that the field is overdamped by the Hubble expansion, and that the kinetic energy is negligible compared with the potential energy. The field ϕ is also assumed to have negligible couplings to all fields other than gravity.

Let us now suppose that there was a period of inflation in the early universe, driven by the potential of some other field. The dynamics of the field ϕ during inflation are strongly influenced by quantum fluctuations, causing different regions of the universe to thermalize with different values of ϕ. Spatial variation of ϕ is thus a natural outcome of inflation.

The probability distribution $\mathcal{P}_*(\phi)$ is determined mainly by the interplay of two effects. The first is the "diffusion" in the field space caused by quantum fluctuations. The dispersion of ϕ over a time interval Δt is $\Delta\phi \sim H(H\Delta t)^{1/2}$, where H is the inflationary expansion rate. The effect of diffusion is to make all values of ϕ equally probable over the interval $\Delta\phi$. The second effect is the differential expansion. Although $V(\phi)$ represents only a tiny addition to the inflaton potential, regions with larger values of $V(\phi)$ expand slightly faster, and thus the probability for higher values of $V(\phi)$ is enhanced. The effect of differential expansion is negligible if (Garriga and Vilenkin, 2000b)

$$V'^2 \gg \rho_{M0}^3 / H^3 M_P^2. \tag{29}$$

In this case, the probability distribution for ϕ is flat in the anthropic range,

$$\mathcal{P}_*(\phi) = \text{const.} \tag{30}$$

The probability distribution for the effective cosmological constant,

$$\rho_V = \rho_\Lambda + V(\phi), \tag{31}$$

is given by

$$\mathcal{P}_*(\rho_V) = \frac{1}{V'} \mathcal{P}_*(\phi), \tag{32}$$

and it will also be very flat, since V' is typically almost constant in the anthropic range.

As we discussed in Sections 4 and 5, a flat prior distribution for the effective cosmological constant in the anthropic range entails an automatic explanation for the two cosmological constant puzzles. On the other hand, if the condition (29) is not satisfied, then the prior probability for the field values with a higher $V(\phi)$ would be exponentially enhanced with respect to the field values at the lower anthropic end. This would result in a prediction for the effective cosmological constant which would be too high compared with observations.

A simple example is given by a potential of the form

$$V(\phi) = \frac{1}{2} \mu^2 \phi^2. \tag{33}$$

We shall assume that $\rho_\Lambda < 0$, so that the two terms in (31) partially cancel one another in some parts of the universe. With $|\rho_\Lambda| \sim (1 \text{ TeV})^4$, the slow roll conditions (27) and (28) give

$$\mu \lesssim 10^{-90} M_P. \tag{34}$$

Thus, an exceedingly small mass scale must be introduced.

The condition (29) yields a lower bound on μ,

$$\mu \gtrsim 10^{-137} M_P. \tag{35}$$

(Here, I have used the upper bound on the expansion rate at late stages of inflation, $H \lesssim 10^{-5} M_P$, which follows from the CMB observations.)

We thus see that models with a variable ρ_V can be easily constructed in the framework of inflationary cosmology. The challenge here is to explain the very small mass scale (34) in a natural way.

6.2. Four-Form Models

Another class of models, first discussed by Brown and Teitelboim (1988), assumes that the cosmological constant is due to a four-form field[2] $F^{\alpha\beta\gamma\delta} = F\epsilon^{\alpha\beta\gamma\delta}$. The field equation for F is $\partial_\mu F = 0$, so F is a constant, but it can change its value through nucleation of bubbles bounded by domain walls, or branes. The total vacuum energy density is given by

$$\rho_V = \rho_\Lambda + F^2/2 \tag{36}$$

and once again it is assumed that $\rho_\Lambda < 0$. The change of the field across the brane is $\Delta F = q$, where the "charge" q is a constant fixed by the model. Thus, F takes a discrete set of values, and the resulting spectrum of ρ_V is also discrete. The four-form model has recently attracted much attention (Banks *et al.*, 2000; Bousso and Polchinski, 2000; Donoghue, 2000; Feng *et al.*, 2000; Garriga and Vilenkin, 2000b) because four-form fields coupled to branes naturally arise in the context of string theory.

In the range where the bare cosmological constant is almost neutralized, $|F| \approx |2\rho_\Lambda|^{1/2}$, the spectrum of ρ_V is nearly equidistant, with a separation

$$\Delta\rho_V \approx |2\rho_\Lambda|^{1/2} q. \tag{37}$$

In order for the anthropic explanation to work, $\Delta\rho_V$ should not exceed the present matter density,

$$\Delta\rho_V \lesssim \rho_{m0} \sim (10^{-3} eV)^4. \tag{38}$$

With $\rho_\Lambda \gtrsim (1TeV)^4$, it follows that

$$q \lesssim 10^{-90} M_P^2. \tag{39}$$

Once again, the challenge is to find a natural explanation for such very small values of q.

To solve the cosmological constant problems, we have to require in addition that (i) the probability distribution for ρ_V at the end of inflation is nearly flat, $\mathcal{P}_*(\rho_V) \approx$ const, and (ii) the brane nucleation rate is sufficiently low, so that the present vacuum energy does not drop significantly in less than a Hubble time. Models satisfying all the requirements can be construcled, but the conditions (i), (ii) significantly constrain the model parameters. For a detailed discussion, see Garriga and Vilenkin (2000b).

[2] The possibility that the cosmological constant could arise as a contribution of a four-form field was first pointed out in Duff and van Nieuwenhuizen (1980).

6.3. Explaining the Small Parameters

Both scalar field and four-form models discussed earlier have some seemingly unnatural features. The scalar field models require extremely flat potentials and the four-form models require branes with an exceedingly small charge. The models cannot be regarded as satisfactory until the smallness of these parameters is explained in a natural way. Here I shall briefly review some possibilities that have been suggested in the literature.

6.3.1. Scalar Field Renormalization

Let us start with the scalar field model. Weinberg (2000) suggested that the flatness of the potential could be due to a large field renormalization. Consider the Lagrangian of the form

$$L = \frac{Z}{2}(\nabla\phi)^2 - V(\phi). \tag{40}$$

The potential for the canonically normalized field $\phi' = \sqrt{Z}\phi$ will be very flat if the field renormalization constant is very large, $Z \gg 1$.

More generally, the effective Lagrangian for ϕ will include nonminimal kinetic terms (Donoghue, 2000; Garriga and Vilenkin, 2000b),

$$L = \frac{1}{2}F^2(\phi)(\nabla\phi)^2 - V(\phi). \tag{41}$$

Take for example $F = e^{\phi/M}$. Then the potential for the canonical field $\psi = Me^{\phi/M}$ is $V(M\ln(\psi/M))$. This will typically have a very small slope if $V(\phi)$ is a polynomial function. It would be good to have some particle physics motivation either for a large running of the field renormalization, or for an exponential function $F(\phi)$ in the Lagrangian (41).

6.3.2. A Discrete Symmetry

Another approach attributes the flatness of the potential to a spontaneously broken discrete symmetry (Dvali and Vilenkin, 2001). The main ingredients of the model are (1) a four-form field $F_{\mu\nu\sigma\tau}$ which can be obtained from a three-form potential, $F_{\mu\nu\sigma\tau} = \partial_{[\mu}A_{\nu\sigma\tau]}$, (2) a complex field X which develops a vacuum expectation value, $\langle X\rangle = \eta e^{ia}$, and whose phase a becomes a Goldstone boson, and (3) a scalar field Φ which is used to break a discrete $Z_{2\mathrm{N}}$ symmetry,

$$Z_{2\mathrm{N}} : \Phi \to \Phi e^{i\pi/N}, \quad a \to -a (\mathrm{or} X \to X^\dagger), \tag{42}$$

Below the symmetry breaking scales of X and Φ, the effective Lagrangian for the model includes a mixing term of the Goldstone a with the three-form potential,

$$g\eta^2 \frac{\langle\Phi\rangle^{\mathrm{N}}}{M_{\mathrm{P}}^{\mathrm{N}}} \epsilon^{\mu\nu\sigma\tau} A_{\nu\sigma\tau} \partial_\mu a. \tag{43}$$

Here, $g \lesssim 1$ is a dimensionless coupling and it is assumed that the Planck scale M_{P} plays the role of the ultraviolet cutoff of the theory.

The effect of the mixing term (43) is to give a mass

$$\mu = g\eta \frac{\langle\Phi\rangle^{\mathrm{N}}}{M_{\mathrm{P}}^{\mathrm{N}}} \tag{44}$$

to the field a. This mass can be made very small if $\langle\Phi\rangle \ll M_{\mathrm{P}}$ and N is sufficiently large. For example, with $\langle\Phi\rangle \sim$ 1 TeV, $\eta \ll M_{\mathrm{P}}$ and $N \geq 6$, we have $\mu \ll 10^{-90} M_{\mathrm{P}}$, as required.

Models of this type can also be used to generate branes with a very small charge. In this case a is assumed to be a pseudo-Goldstone boson, like the axion, and the theory has domain wall solutions with a changing by 2π across the wall. The mixing of a and A couples these walls to the four-form field, and it can be shown that the corresponding charge is

$$q = 2\pi g\eta^2 \frac{\langle\Phi\rangle^{\mathrm{N}}}{M_{\mathrm{P}}^{\mathrm{N}}}. \tag{45}$$

Once again, the anthropic constraint on q is satisfied for $\langle\Phi\rangle \sim$ 1 TeV, $\eta \ll M_{\mathrm{P}}$ and $N \geq 6$.

The central feature of this approach is the $Z_{2\mathrm{N}}$ symmetry (42). What makes this symmetry unusual is that the phase transformation of Φ is accompanied by a charge conjugation of X. It can be shown, however, that such a symmetry can be naturally embedded into a left–right symmetric extension of the standard model (Dvali and Vilenkin, 2001).

6.3.3. String Theory Inspired Ideas

Feng *et al.* (2000) have argued that branes with extremely small charge and tension can naturally arise due to nonperturbative effects in string theory. A potential problem with this approach is that the small brane tension and charge appear to be unprotected against quantum corrections below the supersymmetry breaking scale (Dvali and Vilenkin, 2001). The cosmology of this model is also problematic, since it is hard to stabilize the present vacuum against copious brane nucleation (Garriga and Vilenkin, 2000b).

A completely different approach was taken by Bousso and Polchinski (2000). They assume that several four-form fields F_i are present so that the vacuum energy is given by

$$\rho_V = \rho_\Lambda + \frac{1}{2}\sum_i F_i^2. \tag{46}$$

The corresponding charges q_i are not assumed to be very small, but Bousso and Polchinski have shown that with multiple four-forms the spectrum of the allowed values of ρ_V can be sufficiently dense to satisfy the anthropic condition (38) in the range of interest. However, the situation here is quite different from that in the single-field models. The vacua with nearby values of ρ_V have very different values of F_i, and there is no reason to expect the prior probabilities for these vacua to be similar. Moreover, the low energy physics in different vacua is likely to be different, so the process of galaxy formation and the types of life that can evolve will also differ. It appears therefore that the anthropic approach to solving the cosmological constant problems cannot be applied to this case (Banks *et al.*, 2000).

7. PREDICTION FOR THE EQUATION OF STATE

A generic prediction of models where both CCPs are solved anthropically is that the dark energy equation of state is $P_V = w\rho_V$, where

$$w = -1 \tag{47}$$

with a very high accuracy. In models where ρ_X is the energy density of a four-form field, this equation of state is guaranteed by the fact that the four-form energy density is a constant and can only change by the nucleation of branes (other than that, it behaves exactly like an additional cosmological constant). If ρ_X is a generic scalar field potential, the slow roll conditions (27) and (28) are likely to be satisfied by excess, by many orders of magnitude, rather than marginally. This implies the equation of state (47).

This prediction is similar to the prediction of inflation that the density parameter is $\Omega = 1$. Although it is possible to adjust the inflaton potential so that the requirement of flatness is only marginally satisfied, it is satisfied by a very wide margin in generic models.

I finally note one more anthropic prediction, which is not likely to be tested anytime soon. In all anthropic models, ρ_V can take both positive and negative values, so the observed positive dark energy will eventually start decreasing and will turn negative, and our part of the universe will recollapse to a big crunch. Since the evolution of ρ_V is expected to be very slow on the present Hubble scale, we do not expect this to happen sooner than in a trillion years from now.

8. CONCLUSIONS

I now summarize the predictions that follow from the anthropic approach to the CCP's.

(1) The dark energy equation of state is predicted to be that of the vacuum,

$$P_V = w\rho_V, \tag{48}$$

where $w = -1$ with a very high accuracy. This distinguishes the anthropic models we discussed here from other approaches, such as quintessence (Caldwell *et al.*, 1998; Peebles and Ratra, 1988; Wetterich, 1988; Zlatev *et al.*, 1999) or k-essence (Armendáriz-Picon *et al.*, 2000).

(2) The anthropic predictions for the dark energy density Ω_V and for the Hubble parameter h. are given in Fig. 2. It shows the areas in the $\Omega_V - h$ plane that are excluded at 1σ and 2σ confidence levels. The excluded areas depend on the assumed galactic mass M and on the spectral index n of the density fluctuations. For $M = 10^{12} M_\odot$ the currently popular values $\Omega_V = 0.7$, $h = 0.7$ are marginally excluded at 2σ confidence level for a scale invariant spectrum $n = 1$. Lowering the spectral index relaxes the bounds somewhat. For $h > 0.65$ and $n > .95$, the 1σ prediction is $\Omega_V > 0.79$. These anthropic constraints get weaker when the relevant mass scale M is increased. For example, with $M = 4 \times 10^{12} M_\odot$ a value as low as $\Omega_V = 0.63$ is still allowed at the 2σ level for a scale invariant spectrum. The 1σ prediction in this case is $\Omega_V > 0.78$ (for $h > 0.65$).

(3) Conditions for intelligent life to evolve are expected to arise mainly in giant galaxies that form (or complete their formation) at low redshifts, $z_G \lesssim 1$.

(4) The accelerated expansion will eventually stop and our part of the universe will recollapse, but it will take more than a trillion years for this to happen.

The present bound on the equation of state parameter w from the CMB and supernovae measurements is (Bond *et al.*, 2002) $w < -0.7$, which is consistent with the anthropic prediction of $w = -1$. The value of $w = -1$ is usually associated with a plain cosmological constant. However, if in addition to this equation of state, observations confirm some of the other predictions presented above, this may be taken as an indication that the dark energy is dynamical and that it does take a wide range of values in remote parts of the universe.

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